Atom optics kicked rotor: experimental evidence for a pendulum description of the quantum resonance

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We present measurements of the mean energy for an atom optics kicked rotor ensemble close to quantum resonance. Oscillations in the mean energy in this regime are are shown to be in agreement with a quasi–classical pendulum approximation. The period of the oscillations is shown to scale with a single variable, which depends on the number of kicks.

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The 'kicked rotor' system has provided an extremely useful model for the study of the correspondence between classical and quantum dynamics for almost three decades. This system behaves as a free rotor except during periodically applied momentum kicks, when it experiences a position dependent potential giving rise to sharp changes in momentum. Classically, such a system exhibits essentially stochastic (i.e. *chaotic*) diffusion if the potential strength is large enough [1, 2]. The quantised system, however, avoids this signature of chaos in two principal ways: Firstly, the celebrated phenomenon of *quantum dynamical localisation* may occur in which quantum interference between components of the wavefunction for individual kicked atoms leads to saturation in the energy growth of the ensemble [3, 4]. Secondly, for certain kicking frequencies, quadratic energy growth may occur for atoms with the correct initial momentum in a phenomenon known as *quantum resonance* (QR) [5, 6]. Both of these quantum features are plainly at odds with a classical interpretation, the first because chaotic diffusion is inhibited, and the second because a completely chaotic system cannot be driven on resonance.

In this paper we are concerned with the structure and dynamics of the enhanced energy peaks seen near the quantum resonances of the quantum kicked rotor. For the first time, we present firm experimental evidence of the validity of a quasi-classical pendulum description of the system. In our experiment, we employ an ensemble of laser-cooled atoms which interact with a standing wave of laser light, which is detuned from resonance and pulsed periodically. The AC Stark shift induced by the laser field forms a potential, which depends sinusoidally on the position in the standing wave, leading to dynamics which are formally identical to the kicked rotor. This realisation of the model kicked rotor system is known as the *atom optics kicked rotor* (AOKR)[4]. For certain pulse periods, in this system, enhanced energy peaks arise, as reported in Refs. [7]. Recently, a quasi–classical approximation to the quantum dynamics, which is valid near exact quantum resonance [8] has been developed. This theory, known as ϵ–classical dynamics, accurately predicts the behaviour of kicked atoms near QR [9] utilising a pendulum approximation, which is closely related to, although not equivalent to, the resonance in the classical limit of the quantum kicked rotor [10]. In this paper we measure the oscillations to either side of the main quantum resonance peak that are predicted by the ϵ–classical pendulum approximation. The oscillations are found to move towards the resonance peak with the time dependence predicted by the pendulum scaling of the motion, providing the strongest confirmation yet of the validity of ϵ–classical dynamics in the neighbourhood of the the quantum resonance. The excellent agreement of such details between the ϵ–classical model and a "quantum" feature of the experiment signifies a new level of understanding of quantum dynamics of this simple system.

In a quantum-mechanical picture, the Hamiltonian for the AOKR kicked with period T by an optical standing wave having a wavenumber k_l is given by

$$\mathcal{H}(t') = \frac{\bar{k}\hat{p}^2}{2} + k\cos(\hat{x})\sum_{n=0}^{N}\delta(t'-n) \ , \qquad (1)$$

where $\hat{x}$ is the atomic position operator scaled by $2k_l$, $\hat{p}$ is the momentum operator in units of $2\hbar k_l$, k is the kicking strength, the scaled time is $t' = t/T$ and n is an integer which counts the number of kicks. The quantity $\bar{k}$ may be viewed as a scaled Planck's constant and is defined by the commutator relation $[\hat{x}, \hat{\rho}] = i\bar{k}$, where $\bar{k} = 8\omega_r T$ (ω_r is the frequency associated with the energy change after a single photon recoil for Rubidum for a photon with $\lambda = 780$nm). We may write the momentum as $\hat{p} = (\hat{n} + \beta)$, where $\hat{n}$ the integer momentum operator and β the quasimomentum (or noninteger part of the momentum) which is conserved during kicking. The δ–kick approximation is quite good in the experiments described in this paper as the pulse width was 320ns compared with a period of $\approx 30\mu$s.

We now present a brief summary of the ϵ–classical theory of the quantum resonance peaks as it applies to our experiments, following the notation of Refs. [8]. The Floquet operator for our system is [5, 8, 11]

$$\hat{U} = \exp(\mathrm{i}k\cos\hat{\hat{x}})\exp(-\mathrm{i}\bar{k}\hat{p}^2/2). \qquad (2)$$

In Ref. [8] it was shown that the evolution operator (2)

could be written in an equivalent form:

$$\hat{U}_\beta = \exp\left(-\mathrm{i}\frac{\tilde{k}}{|\epsilon|}\cos(\hat{\theta})\right)\exp\left(-\frac{\mathrm{i}}{|\epsilon|}\hat{H}_\beta\right), \qquad (3)$$

where $\epsilon = \bar{k} - 2\pi\ell$ for integer ℓ, $\tilde{k} = |\epsilon|\, k$ and $\hat{H}_\beta$ is a function of β and the new momentum operator $\hat{I} = |\epsilon|\,\hat{n}$ given by $\hat{H}_\beta = \frac{1}{2}\mathrm{sign}(\epsilon)\hat{\mathrm{I}}^2 + \hat{\mathrm{I}}(\pi\ell + \bar{\mathrm{k}}\beta)$. We see that in Eq. 3, the quantity $|\epsilon|$ plays the part of Planck's constant. However, $|\epsilon| \to 0$ is a fictitious classical limit which corresponds to approaching exact quantum resonance. For $\bar{k}$ close to $2\pi\ell$, therefore, the quantum dynamics is approximated well by the *classical* map [8]

$$\begin{aligned} J_{t+1} &= J_t + |\epsilon|\, k\sin(\vartheta_t), \\ \vartheta_{t+1} &= \vartheta_t + J_{t+1}, \end{aligned} \qquad (4)$$

where $J = \pm I + \pi\ell + \tau\beta$ and $\vartheta = \theta + \pi(1 - \mathrm{sign}(\epsilon))/2$ with the mean energy given by $\langle E\rangle = |\epsilon|^{-2}\left\langle (J - J_0)^2/2\right\rangle$.

In the limit as $|\epsilon| \to 0$, Wimberger *et al.* showed that the mean energy of a kicked atomic ensemble is $\langle E_t\rangle = k^2 t/4$. For $|\epsilon| > 0$, a pendulum approximation to the dynamics of the map 4 is appropriate [2], with the pendulum Hamiltonian $H_{\mathrm{pend.}} = J^2/2 + |\epsilon|\, k\cos(\vartheta)$. The pendulum motion then has a characteristic resonance period $t_{\mathrm{res}} = 1/\sqrt{k\,|\epsilon|}$. The off–resonant mean energy, when scaled by the peak energy $\langle E_{t,\epsilon}\rangle = k^2 t/4$ may be written in terms of $x = t/t_{\mathrm{res}}$ as [13]

$$\frac{\langle E_{t,\epsilon}\rangle}{\langle E_{t,\epsilon=0}\rangle} \approx (1 - \Phi_0(x)) + \frac{4}{\pi x}G(x), \qquad (5)$$

where the term $G(x)$ is the energy due to the pendulum motion. The term $1 - \Phi_0(x)$ is the energy due to the remaining phase space which is not affected by the pendulum resonance. It decays rapidly for $t/t_{\mathrm{res}} \lesssim 1$ (see Fig. 1)and thus for $x \gg 1$ we may write

$$\frac{\langle E_{t,\epsilon}\rangle}{\langle E_{t,\epsilon=0}\rangle} \approx \frac{4}{\pi x}G(x). \qquad (6)$$

The function $G(x)$ is given explicitly by

$$G(x) = \frac{1}{8\pi}\int_0^{2\pi} \mathrm{d}\theta \int_{-2}^{2} \mathrm{d}J_0' J'(x, \theta_0, J_0')^2, \qquad (7)$$

where J' is the general solution for the pendulum momentum in terms of the Jacobi elliptic functions (see, for example, Ref. [12])). The function $G(x = t/t_{\mathrm{res}})$ is shown in Fig. 1. We see that it consists of decaying oscillations, which appear at constant values of x. Whilst the first local maximum of G is not visible in scans of the quantum resonance peak, due to the predominance of the term $1 - \Phi_0(x)$, the second maximum at constant $x = x_0 \approx 11.8$ should be visible as oscillations to either side of the principle quantum resonance peak. Furthermore, since these oscillations have a constant x position,

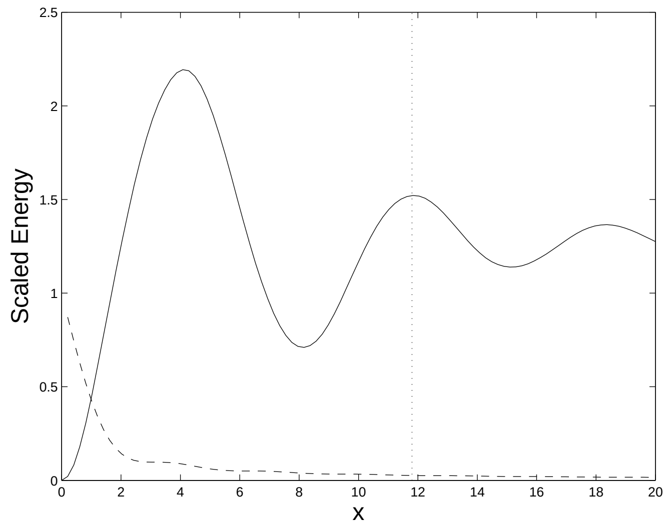

FIG. 1: The function $G(x)$ as given by Eq. 7. Note the secondary maximum at $x \approx 11.8$ (marked by the dotted line) which is responsible for the structure to either side of quantum resonance. For larger x the function saturates to a value $\alpha \approx 1.3$. The dashed line shows $1 - \Phi_0(x)$.

their position relative to the quantum resonance should change as a function of time according to the equation

$$|\epsilon| = \frac{x_0}{t^2 k}. \qquad (8)$$

To confirm the validity of this classical pendulum approximation near quantum resonance, we perform a kicked rotor experiment using ultracold rubidium 85 atoms from a standard magneto-optical trap [14], which is loaded from a background vapour. After an accumulation phase, the quadrupole magnetic field is turned off and the trap lasers are switched to a larger detuning for a 3 ms cooling phase, after which the trap lasers are extinguished. The temperature of the atom cloud at that moment is $\sim$ 10 μK. The atoms are then subjected to a periodic sequence of kicks by the (linearly polarised) standing wave laser field. The atomic ensemble is subsequently allowed to expand for 15 ms, after which the ensemble fluorescence is imaged on a CCD camera (Apogee AP47p) by flashing on the molasses lasers for 5 ms. During the imaging period, the atoms experience an 'optical molasses' and hence do not move significantly. We obtain the positional variances of the atomic cloud, in both the direction of the kick laser ('kicked') and orthogonal to that ('non-kicked'), numerically from the image and convert these to average velocity squared, and from that to kinetic energy. We take the energy in the non-kicked direction as the initial energy. The experiment is then repeated for different kick periods or kick numbers. The average initial energy is determined from an experimental run over a range of parameters, and is used to rescale the energy ratio to an energy.

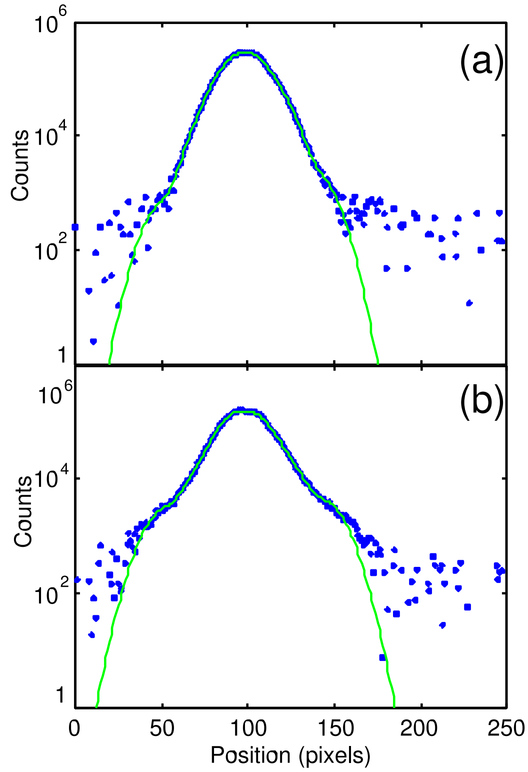

FIG. 2: Momentum distributions after 14 kicks, for kick periods of (a) 30.5 μs (b) and 32.5 μs, corresponding to $\bar{k} = 5.9$ and $\bar{k} = 6.3$ respectively. The experimental data is shown as discrete (points). The solid lines and the result of ϵ-classical simulations. These results are for a scaled kick strength $k =$??.

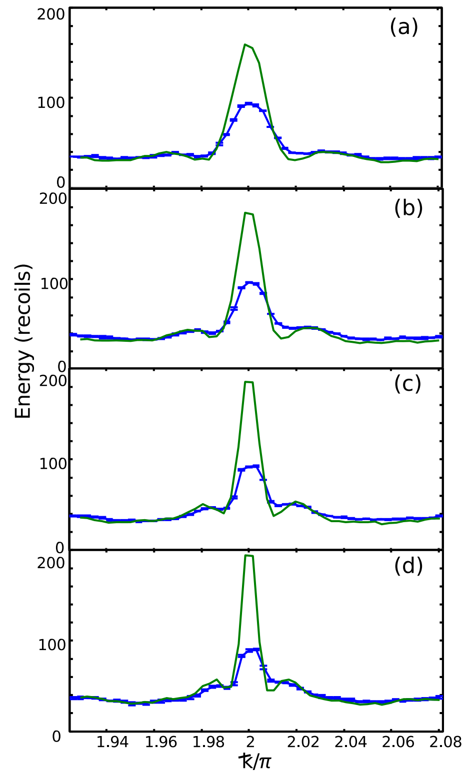

FIG. 3: Measured mean energies (discrete points) as a function of the kick period for (a) 12 kicks, (b) 14 kicks, (c) 16 kicks and (d) 18 kicks. Solid lines correspond to ϵ-classical simulations. The scaled kick strength $k = 4.2$.

The kicking laser beam was obtained from a Toptica DLX110 laser system, which was locked to the D_2 transition in ^{87}Rb, obtaining a laser detuning of 1.3 GHz from the $F = 3 \rightarrow F = 4$ transition in ^{85}Rb. The beam was passed trough an AOM, which was controlled by a home-built programmable pulse generator, allowing us to pulse the laser with adjustable pulse number, period and amplitude. The pulse duration for most experiments was set to 320 ns. The rise and fall times of the laser pulse were less than 50 ns. To spatially filter the laser beam, it was then passed through a polarisation preserving single mode fibre. After collimation, the radius ($1/e^2$) of the gaussian laser beam was 3.0 mm. The laser beam was passed through a polarising beam splitter to ensure linear polarisation. The large detuning and linear laser polarisation yield equal kick strengths for all magnetic sublevels of the $F = 3$ ground state. The maximum kick laser power was 100 mW, which for a 320 ns pulse and a 1.3 GHz laser detuning yields a scaled kick strength $k = 5.2$. Larger kick strengths could be obtained by lengthening the kick pulse. The detuning of the kick laser yielded a negligible spontaneous emission rate for all laser powers used. This was verified by observing the cloud energies for a range of kick numbers, up to 80 kicks, which showed negligible energy growth after a certain number of kicks. The initial radius of the atomic cloud was 0.3 mm (1σ), significantly smaller than the size of the kicking laser beam.

In figure 2 the momentum distribution of the atoms is displayed after 14 kicks, for a kicking period (a) close to to the first quantum resonance ($\bar{k} = 2\pi$) and (b) exactly on resonance. For the on resonant case, a pedestal is seen in the wings of the central peak corresponding to a small population of resonant atoms. Experimentally, it is difficult to resolve the energy due to these atoms but their relatively large momenta means that they contribute greatly to the total mean energy. This lack of resolution (also noted by other groups [9]) causes the experimental values for the energy on the quantum resonance to be smaller than the theoretical values (see Fig. 3). For off resonant kicking periods the pedestal is not present, and the experimental and simulated values of the mean energy show good agreement.

In figure 3, the measured mean energy is displayed as a function of the kicking period around the first primary quantum resonance, for different numbers of kicks. It is interesting that an asymmetry in the peaks is found (also observed in [9]). This feature is most likely due to a

small amount of spontaneous emission occuring with each pulse. In fact, quantum simulations including the effect of spontaneous emission reproduce this effect. The assymetry is, therefore, a purely experimental effect which is not allowed for by the ϵ–classical theory, but which does not destroy the basic structure we seek to measure.

The experimental measurements are compared with epsilon classical simulations. We find good agreement except near exact resonance where the mean energy calculation is affected by a lower signal–to–noise ratio for the small population of resonant atoms. However the structure of interest in these experiments lies away from the central resonance peak. In particular, we draw attention to the small side peaks, which we are resolved accurately here for the first time. These secondary maxima are expected to exist if the pendulum scaling law (6) is correct. Furthermore, as shown in Figure 4, the side peaks move towards the quantum resonance as the number of kicks increases exactly as predicted by Eq. 8 (choosing $x_0 = 11.2$ to account for the fact that G is divided by x in the energy scaling). We would like to emphasise the very accurate nature of the measurements performed here. Fig. 4 demonstrates that there is quantitative agreement between the position of the side peaks as a function of kick number t and Eq. 8, providing strong evidence that a classical pendulum scaling of the dynamics is valid in this regime. These results show that the function $G(x)$ is an excellent predictor of the dynamical evolution of the atom ensemble near quantum resonance

This confirmation of Eq. 8 is also a direct way of demonstrating that the resonance peak becomes narrower in a sub–Fourier manner, that is, at a rate faster than $1/t$. This feature was also demonstrated in [9] in [15] for other types of quantum resonances.

In conclusion, we have made the first careful measurements of the oscillations near quantum resonance, which are predicted by a classical pendulum scaling law of the near resonant dynamics. The structure itself is well resolved in our experiments. Furthermore, the motion of the oscillations with kick number has been observed to agree very well with the predictions of the ϵ–classical theory. These measurements set a new level of accuracy in the measurement of the rather subtle near resonant effects predicted by this new semi–classical approach to the dynamics at quantum resonance.

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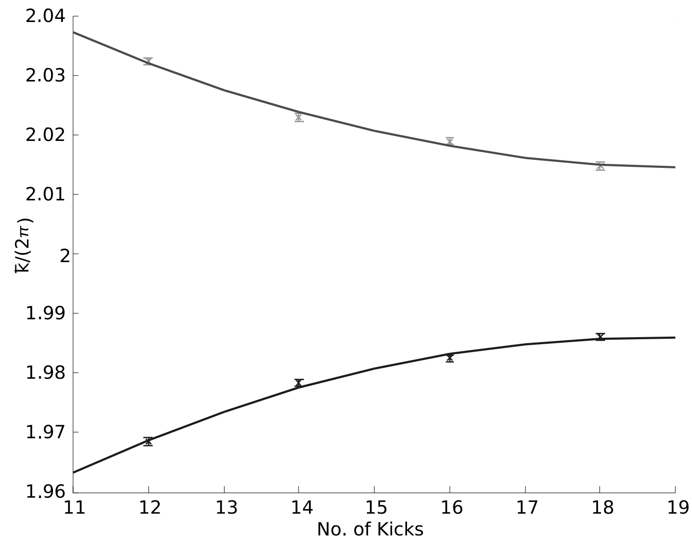

FIG. 4: Peak position of the near resonant peaks plotted against kick number. The solid curve is that given by Eq. 8 for $k = 4.1$. We see excellent agreement between the predictions of the ϵ–classical scaling function and the observed movement of the peaks as N increases.

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